HYPER VERSION

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Brane surgery with Thom classes

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Abstract: We propose a method to investigate the conservation of brane charges at the intersection of two or more branes using the Thom classes of their normal bundles. In particular we find a relation between the charge of the branes involved in the configuration and the charge of the defects on the branes due to the intersection. We also explore the applications of our method for various brane intersections in type II strings and M-theory.

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1. Introduction

One of the most remarkable properties of branes is that they can end or intersect with others [1, 2, 3, 4]. The arguments that lead us to believe that this is the case are either based on brane charge conservation [2, 5] or the analysis of the couplings of brane effective theories. In same cases, this can also be achieved by direct construction like in the case of fundamental strings which can end on D-branes (for a review see [6]). All these arguments are of course related. The conservation of brane charge method has been tackled by Townsend using a deformation argument which is known as brane surgery [5]. This argument is based on the existence of Chern-Simons terms in supergravity theories, the use of their field equations and certain assumptions about the behaviour of the supergravity gauge potentials near the intersection of two or more branes. One of the advantages of brane surgery is that it applies for branes in a curved background. In the context of effective brane worldvolume theories, the brane intersections or boundaries are thought in a way similar to the interactions of particles in standard quantum field theory. So, the conservation of brane charges in this approach corresponds to the conservation of particle charges at the interactions vertices of the effective brane theory.

From the (worldvolume) perspective of the branes involved in an intersection or a boundary, the intersection or the boundary is described by a defect. This defect can be viewed as a soliton like object and its charge can be measured on each brane involved in the configuration [4]. One expects that the charges of the defects and the charges of the branes of the configuration are related amongst themselves. To derive such relations, one first uses the fact that these defects have a brane interpretation so they are usually called worldvolume branes or brane worldvolume solitons. The brane surgery method of [5] then provides a relation amongst the charges of the defects and the charges of the branes of the configuration.

In this paper, we shall propose another way to describe the brane charge conservation at the intersection or boundary of two or more branes. Our construction is based on the Thom classes $\phi(E)$ of vector bundles E and their properties. In particular, we view p-branes as (p+1)-dimensional submanifolds B_p in the spacetime M which asymptotically behave as their associated supergravity solutions. Their charge is then computed by integrating an appropriate form field strength at a sphere at infinity. We shall see that this is the same as evaluating the Thom class $\phi(N_{(p)})$ of the normal bundle $N_{(p)}$ of B_p in the spacetime M; this has also been proposed by Witten in [7]. Next using the properties of Thom classes, if two branes B_p and B_q intersect on $Z_k = B_p \cap B_q$, ($\mathrm{dim} Z_k = k+1$), we shall establish the relation

$$Q_{(p)} Q_{(k\hookrightarrow p)} = Q_{(q)} Q_{(k\hookrightarrow q)} \,, \tag{1.1}$$

where $Q_{(p)}$ is the charge of the p-brane, $Q_{(q)}$ is the charge of the q-brane, $Q_{(k\hookrightarrow p)}$ is the charge of the k-brane defect Z_k on the p-brane and $Q_{k\hookrightarrow q}$ is the charge of Z_k on the q-brane. The charges $Q_{(k\hookrightarrow p)}$ and $Q_{(k\hookrightarrow q)}$ are associated with the Thom classes of the normal bundles of Z_k in B_p and B_q, respectively. The above relation among the various charges has two implications the following:

(i) The charges of the k-brane defect as measured on the p-brane and on the q-brane are proportional to the number of q-branes and the number of p-branes involved in the configuration, respectively.

(ii) There is a relation between the units that one measures the charges of the defects with those of the p-and q-branes.

We shall explore the relation (1.1) amongst the charges for various boundaries and brane intersections in strings and M-theory. In particular, we shall find that the unit of the charge of the boundary of a fundamental string ending on a D-brane is proportional to the fundamental string tension. Similarly, the unit of the charge of a M-2-brane ending on a M-5-brane is proportional to the M-2-brane tension. We shall also find that in some cases the charges of the defects on intersecting branes are related to the Euler number of certain vector bundles. In addition, our method

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can be used to investigate charge conservation for intersections that involve three or more kinds of branes; we shall demonstrate this for a triple M-brane intersection.

This letter has been organized as follows: In section two, we use Thom classes to describe brane charge conservation for intersecting branes and derive the equation (1.1). In section three, we apply our formalism to investigate the conservation of charges in type II intersections. In section four, we investigate charge conservation in M-brane intersections, and in section five we present our conclusions and remark on the application of our results in the context of supergravity solutions with the interpretation of intersecting branes.

2. Brane Charges and Thom Classes

2.1 The p-brane charge

Spacetimes with a p-brane interpretation have an asymptotic region which is isomorphic to $\mathbb{R}^{(1,p)} \times \mathbb{R}^{d-p-1}$, where $\mathbb{R}^{(1,p)}$ are the worldvolume directions and $\mathbb{R}^{d-p-1}$ are the transverse or normal directions of the p-brane [8]. This is the so called spatial transverse infinity which can be thought of as the spatial infinity of the spacetime far away from the location of the brane. Any p-brane in a d-dimensional spacetime has an associated $(d-p-2)$-form field strength F. The charge Q_p per unit volume (in some frame) of the p-brane can be computed by evaluating F at a sphere $S^{d-p-2} \subset \mathbb{R}^{d-p-1}$ at infinity as

$$Q_p = \frac{1}{\mathrm{Vol}(S^{d-p-2})} \int_{S^{d-p-2}} F \,, \tag{2.1}$$

where $\mathrm{Vol}(S^{d-p-2})$ is the volume of the unit S^{d-p-2} sphere. Typically, the asymptotic behaviour of F in some angular coordinates in $\mathbb{R}^{d-p-1}$ is

$$F = -\frac{Q_p}{(d-p-3)} \star d \frac{1}{r^{d-p-3}} \tag{2.2}$$

as $r \to \infty$, where r is the radius and the Hodge star is that on $\mathbb{R}^{d-p-1}$.

An alternative way to compute the charge of a brane is to observe that

$$dF = Q_p \delta(r) \,, \tag{2.3}$$

where δ is a $(d-p-1)$-form with support at $r = 0$. So we can compute Q_p by integrating dF over $\mathbb{R}^{d-p-1}$, i.e.

$$Q_p = \int_{\mathbb{R}^{d-p-1}} dF \,. \tag{2.4}$$

There is a geometric way to view this calculation. For this let us identify the p-brane as a submanifold B_p of the spacetime M. The normal bundle $N_{(p)}$ of B_p in M is defined as

$$TB_p \to TM|_{B_p} \to N_{(p)} \,. \tag{2.5}$$

So at the transverse spatial infinity, $N_{(p)} = \mathbb{R}^{(1,p)} \times \mathbb{R}^{d-p-1}$. The form dF can be thought of as a $(d-p-1)$-form on $N_{(p)}$ which has support at the zero section of $N_{(p)}$. In addition, the computation of charge of the p-brane above can be thought of as the integration of dF along a fibre $\mathbb{R}^{d-p-1}$ of $N_{(p)}$.

For every rank n vector bundle E over a manifold S, the Thom class,[1] $\phi(E)$, of E is a (smooth) n-form in $\phi(E)$ which has the following properties:

(i) The integration of $\phi(E)$ along any fiber of E gives one.

(ii) $\phi(E)$ has support very close to the zero section of E.

We shall not give the details of the construction of the Thom class $\phi(E)$ of E. This is explained in [9]. The Thom class of the normal bundle $N_{(p)}$ of a submanifold B_p in M is the Poincaré dual of B_p.

Now we shall take dF to be in the (cohomology) class of $\phi(N_{(p)})$ of the normal bundle $N_{(p)}$ of the p-brane B_p in the spacetime M. For this we appropriately rescale $\phi(N_{(p)})$ such that integration of $\phi(N_{(p)})$ over the fibers of $N_{(p)}$ gives at transverse spatial infinity the charge Q_p of the p-brane. The use of the Thom class $\phi(N_{(p)})$ instead of dF to compute the charge[2] of a p-brane has two advantages the following:

(i) The charge per unit volume of a p-brane can be computed not only at the transverse spatial infinity but at any point on the brane B_p.

(ii) The Thom class $\phi(N_{(p)})$ need not satisfy the supergravity field equations. Instead it is sufficient to assume that there is a representative in the class of $\phi(N_{(p)})$ that obeys the supergravity field equations.

The latter point allows the computation of the charges at brane intersections to be independent from the details of the dynamics. In the remaining sections, we shall use the properties of the Thom classes to investigate the charges of worldvolume brane defects of intersecting branes.

2.2 Intersecting branes

The typical set up of an intersecting brane configuration is that of a p-brane, B_p, and and a q-brane, B_q, intersecting on a k-brane $Z_k = B_p \cap B_q$ in a spacetime M. Now since the defect Z_k has a brane interpretation as viewed from the worldvolume perspective of both the p-brane and the q-brane, we shall use the Thom classes of the normal bundles of Z_k in B_p and in B_q to compute its charges. For this, let $\phi(N_{(p)})$ and $\phi(N_{(q)})$ be the Thom classes of the normal bundles $N_{(p)}$ and $N_{(q)}$ of the p-brane

[1] We assume that the manifolds and the vector bundles involved are oriented.

[2] One may wonder whether it is possible instead of the Thom class of N_p that is associated with dF to use another class on N_p that is associated with F for the computation of the charge everywhere on a p-brane. However, no such class exists for N_p unless its Euler class vanishes [9].

and the q-brane in the spacetime M, respectively. Far away from the q-brane, the charge Q_p of the p-brane can be computed as in the previous section by integrating the Thom class $\phi(N_{(p)})$ along a fibre of the normal bundle $N_{(p)}$. Since this can be done at any point in B_p, we can also evaluate the charge of the p-brane at a point in the intersection Z_k. However at the intersection Z_k of the p-brane with the q-brane, the normal bundle of the p-brane splits[3] as

$$N_{(p)}|_{Z_k} = N_{(p+q)}|_{Z_k} \oplus N_{(k\hookrightarrow q)}\,, \tag{2.6}$$

where $N_{(p+q)} = N_{B_p \cup B_q}$ is the normal bundle[4] of B_p and B_q in the spacetime M, and $N_{(k\hookrightarrow q)}$ is the normal bundle of Z_k in B_q. This decomposition of $N_{(p)}|_{Z_k}$ can be seen by observing that Z_k is a submanifold of B_q and so $N_{k\hookrightarrow q}$ is a subbundle of $N_{(p)}|_{Z_k}$. Using the above splitting of $N_{(p)}$ and the properties of Thom classes, we can write

$$\phi(N_{(p)}|_{Z_k}) = \phi(N_{(p+q)}|_{Z_k}) \wedge \phi(N_{(k\hookrightarrow q)})\,. \tag{2.7}$$

In turn, this implies that

$$Q_{(p)} = Q_{(p+q)} Q_{(k\hookrightarrow q)}\,, \tag{2.8}$$

where $Q_{(p+q)}$ is interpreted as the charge of the "whole configuration" and $Q_{(k\hookrightarrow q)}$ is interpreted the charge of the k-brane worldvolume defect from the perspective of the q-brane. Repeating the same argument for the q-brane, we find that

$$Q_{(q)} = Q_{(p+q)} Q_{(k\hookrightarrow p)}\,, \tag{2.9}$$

where $Q_{(k\hookrightarrow p)}$ is the charge of the k-brane worldvolume defect from the perspective of the p-brane. Eliminating $Q_{(p+q)}$, we find

$$Q_{(p)} Q_{(k\hookrightarrow p)} = Q_{(q)} Q_{(k\hookrightarrow q)}\,. \tag{2.10}$$

We have now derived the equation (1.1) of the introduction. The charge $Q_{(p)}$ can be written as

$$Q_{(p)} = \mu_{(p)} n_p\,, \tag{2.11}$$

where $\mu_{(p)}$ is the unit of charge of the p-brane and $n_p \in \mathbb{Z}$ is the number of p-branes of the configuration. Similarly, we can write $Q_{(q)} = \mu_{(q)} n_q$ for the q-brane. The equation (2.10) is valid for any number of p- and q-branes. This implies that

$$\begin{aligned} Q_{(k\hookrightarrow p)} &= \mu_{(k\hookrightarrow p)} n_q \\ Q_{(k\hookrightarrow q)} &= \mu_{(k\hookrightarrow q)} n_p\,, \end{aligned} \tag{2.12}$$

and

$$\mu_{(p)} \mu_{(k\hookrightarrow p)} = \mu_{(q)} \mu_{(k\hookrightarrow q)}\,, \tag{2.13}$$

[3] More precisely, we have $N_{(k\hookrightarrow q)} \to N_{(p)}|_{Z_k} \to N_{(p+q)}|_{Z_k}$.

[4] The fibre directions of this bundle are the overall transverse directions of an intersecting brane configuration in the terminology of [4].

where $\mu_{(k\hookrightarrow p)}$ and $\mu_{(k\hookrightarrow q)}$ are the units of charges of the k-brane worldvolume defects on the p-brane and the q-brane, respectively. From these it is clear that the charge of the k-brane defect on the p-brane is proportional to the number of q-branes, and similarly the charge of the k-brane defect on the q-brane is proportional to the number of p-branes of the configuration. This in fact is what one naively expects in the context of defects which are associated with brane intersections.

The above computation can be easily extended to intersections that involve more than two kinds of branes. We shall not attempt to give the general analysis here. Instead, we shall find the relations amongst the charges associated with a triple M-brane intersection in an example below.

3. Intersections in string theory

All the brane intersections in string theory can be derived from brane intersections in M-theory using S- and T-dualities. Nevertheless, some features of brane intersections can be better understood in the context of string theory. We shall be concerned with the class of intersections which involves two kinds of branes. Our intention is not to give a complete treatment of all possible brane intersections in string theory but it is rather limited to present some examples of the method. From these it will become clear how one can investigate any brane intersection in this way.

3.1 Fundamental strings ending on D-branes

One of the most well studied boundaries is that of a fundamental string ending on a D-p-brane for $p \leq 6$. Using (2.10) for $p = p$ and $q = 1$, we find that

$$Q_{(p)}Q_{(0\hookrightarrow p)} = Q_{(0\hookrightarrow 1)}Q_{(1)}\,. \tag{3.1}$$

For $p = 0$, the Thom class of $N_{(0\hookrightarrow 0)}$ can be taken to be the zero form which implies that the charge $Q_{(0\hookrightarrow 0)}$ vanishes. The equation (3.1) remains consistent provided $Q_{(0\hookrightarrow 1)} = 0$ and so one concludes that within this formalism fundamental strings cannot end on a D-0-brane in agreement with [5]. The equation (3.1) for $p = 1$ adapted for the units of charges reads[5]

$$\mu_{(1_D)}\mu_{(0\hookrightarrow 1_D)} = \mu_{(0\hookrightarrow 1_{NS})}\mu_{(1_{NS})}\,. \tag{3.2}$$

This is the case of a string junction [10]. It is natural to identify $\mu_{(0\hookrightarrow 1_D)}$ with the unit of charge of the Born-Infeld field of the D-string. To find the unit of this charge, we first note that the D-string tension[6] $T_{(1_D)}$ in the presence of constant Born-Infeld

[5] We shall add the subscripts NS and D with the obvious interpretation whenever it is necessary to avoid confusion.

[6] We use the string frame relation $T_{(p)} = g_s^a \mu_{(p)}$ between the tension $T_{(p)}$ and the unit of charge $\mu_{(p)}$ of a p-brane, where g_s is the string coupling, $a = 0$ for fundamental string, $a = -1$ for D-p-branes and $a = -2$ for NS-5-branes. In the conventions of [13], $\mu_{(p)} = (2\pi)^{-p}(\alpha')^{-\frac{p+1}{2}}$.

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field $F = F_{01}$ changes as

$$T_{(1_D)} + \frac{1}{2}\frac{\mu_{(1_{NS})}}{g_s}(2\pi\alpha')^2 F^2 . \tag{3.3}$$

This follows from the Born-Infeld action of a D-string; a similar calculation has been done in [11] for $\alpha' = 1$. On the other hand the tension of a state involving a D-string and a NS-string is

$$\mu_{(1_{NS})}\left(1 + g_s^{-2}\right)^{1/2} = \left(\frac{1}{g_s} + \frac{1}{2}g_s\right)\mu_{(1_{NS})} + O(g_s^3) . \tag{3.4}$$

From the above two equations at small string coupling, we find that

$$\mu_{(0\hookrightarrow 1_D)} = F = g_s T_{(1_{NS})} . \tag{3.5}$$

Substituting this into the equation (3.2), we get

$$\mu_{(0\hookrightarrow 1_{NS})} = g_s T_{(1_{NS})} . \tag{3.6}$$

So the defect on the fundamental string has unit charge proportional to its tension. This may have been expected since the defect is a domain wall [12].

For the rest of the cases, we remark that the units of charges of D-p-branes [6, 14, 13, 15] are related by the equation

$$\mu_{(p)} = \frac{1}{2\pi}\mu_{(1)}\mu_{(p-2)} . \tag{3.7}$$

Using this equation and (2.13) for $p = p$, $q = 1$ and $k = 0$, we find that

$$\mu_{(0\hookrightarrow 1)} = \frac{1}{2\pi}\mu_{(p-2)}\mu_{(0\hookrightarrow p)} . \tag{3.8}$$

We proceed by observing that the right hand side of (3.2) is T-duality invariant provided that it is performed in directions orthogonal to the string. However, the left hand side changes according to the familiar T-duality rules for D-branes (for the T-duality rules of the defects see [16]). Moreover, the fact that in all cases the defect on the fundamental string is a domain wall suggests that (3.6) is valid for $0 < p \leq 6$. Using these, we find that $\mu_{(0\hookrightarrow p)} = 2\pi g_s \mu_{(1_{NS})}\mu^{-1}_{(p-2)}$. This expression for $\mu_{(0\hookrightarrow p)}$ can also be directly computed in a way similar to that for $p = 1$ above.

3.2 Three-brane solitons in type IIA

In type IIA, (i) two (non-parallel) NS-5-branes[7] and (ii) a NS-5-brane and a D-4-brane intersect on a three-brane soliton defect. We shall postpone the investigation of (i) as well as that of the associated via T-duality intersection in type IIB for later. This is because these intersections are similar to that of two M-5-branes

[7] Cancellation of anomalies in the effective theory of NS-5-branes requires that the Euler number of the normal bundle of IIA NS-5-branes vanishes [7].

intersecting on a three-brane. So all the details will follow from the investigation of this intersection in M-theory.

Applying (2.10) for the intersection of a NS-5-brane and a D-4-brane on a three-brane, we find

$$Q_{(5)}Q_{(3\hookrightarrow 5)} = Q_{(3\hookrightarrow 4)}Q_{(4)}\,. \tag{3.9}$$

The unit of charge of the NS-5-brane and with that of D-4-brane are related [14, 15] by

$$\mu_{(5)} = \frac{1}{2\pi}\mu_{(0)}\mu_{(4)}\,. \tag{3.10}$$

Substituting this equation into (2.13) for $p = 5$, $q = 4$ and $k = 3$, we get

$$\frac{1}{2\pi}\mu_{(0)}\mu_{(3\hookrightarrow 5)} = \mu_{(3\hookrightarrow 4)}\,. \tag{3.11}$$

If in addition we take $\mu_{(3\hookrightarrow 4)} \sim \mu_{(4)}$, because the 3-brane defect is a domain wall in the D-4-brane, then $\mu_{(3\hookrightarrow 5)} \sim \mu_{(4)}\mu_{(0)}^{-1}$.

3.3 Two-brane solitons in type IIB

In type IIB, a D-3-brane intersects with a NS-5-brane on a 2-brane. Using (2.10) for this intersection, we find that

$$Q_{(5_{NS})}Q_{(2\hookrightarrow 5_{NS})} = Q_{(2\hookrightarrow 3_D)}Q_{(3_D)}\,. \tag{3.12}$$

The unit of charge of IIB NS-5-brane and that of D-string are related [14, 15] as

$$\mu_{(5_{NS})} = \frac{1}{2\pi}\mu_{(1_D)}\mu_{(3_D)}\,. \tag{3.13}$$

Substituting this equation into (2.13) for $p = 5$, $q = 3$ and $k = 2$, we get

$$\frac{1}{2\pi}\mu_{(1_D)}\mu_{(2\hookrightarrow 5_{NS})} = \mu_{(2\hookrightarrow 3_D)} \tag{3.14}$$

and thus we establish that the unit charge of 2-brane defect on the NS-5-brane and the unit of charge of 2-brane defect on the D-3-brane are related via the unit of charge of D-string.

Finally in type IIB a NS-5-brane and a D-5-brane intersect on a 2-brane. For this intersection, (2.10) becomes

$$Q_{(5_{NS})}Q_{(2\hookrightarrow 5_{NS})} = Q_{(2\hookrightarrow 5_D)}Q_{(5_D)}\,. \tag{3.15}$$

The unit of charge of NS-5-brane and that of D-5-brane are related [14, 15] as

$$\mu_{(5_{NS})} = \frac{1}{2\pi}\mu_{(-1_D)}\mu_{(5_D)}\,, \tag{3.16}$$

which upon substitution in (2.13) for $p = 5$, $q = 5$ and $k = 2$ leads to

$$\frac{1}{2\pi}\mu_{(-1_D)}\mu_{(2\hookrightarrow 5_{NS})} = \mu_{(2\hookrightarrow 5_D)}\,, \tag{3.17}$$

where $\mu_{(-1_D)}$ is the unit of charge of IIB instanton.

4. M-branes

The are two types of brane intersections [4, 17] in M-theory.[8] The first is that of a membrane ending on a M-5-brane with defect a (self-dual) string. The other is that two non-parallel M-5-branes intersecting on a 3-brane. The latter intersection is part of a more general intersection rule which states that two non-parallel p-branes intersect on a (p-2)-brane. There is a large number of triple M-brane intersections. Here we shall not do a systematic investigation of triple intersections. Instead, we shall examine the case of two (non-parallel) membranes ending on a M-5-brane.

4.1 Membranes ending on M-5-branes

The defect on a M-2-brane ending on a M-5-brane is a string. This defect from the perspective of the M-5-brane is the self-dual string while from the perspective of the M-2-brane is a boundary. Applying (2.10) in this case for $p = 5$, $q = 2$ and $k = 1$, we find

$$Q_{(5)} Q_{(1 \hookrightarrow 5)} = Q_{(2)} Q_{(1 \hookrightarrow 2)} \, . \tag{4.1}$$

Using the relation $\mu_{(5)} = \frac{1}{2\pi} (\mu_{(2)})^2$ of the unit of charge of the M-5-brane[9] and that of M-2-brane [14, 15], we can rewrite (2.13) adopted to this case as

$$\mu_{(1 \hookrightarrow 2)} = \frac{1}{2\pi} \mu_{(2)} \mu_{(1 \hookrightarrow 5)} \, . \tag{4.2}$$

Therefore, the unit of charge $\mu_{(1 \hookrightarrow 2)}$ of the defect on the M-2-brane is proportional to the unit of charge charge of the M-2-brane as it is expected for a domain wall. We remark that in this method the charge of the string defect on the M-5-brane is not naturally associated with a self-dual three-form as it may have been expected. However since the value of the charge depends only the cohomology class of the form field strengths, there may be a self-dual representative of this class.

4.2 The 3-brane worldvolume soliton

The relation (2.10) of the various charges in the case of two non-parallel M-5-brane intersecting on a 3-brane defect is

$$Q_{(5)} Q_{(3 \hookrightarrow 5)} = Q'_{(5)} Q'_{(3 \hookrightarrow 5)} \, , \tag{4.3}$$

where $Q_{(5)}$ and $Q'_{(5)}$ are the charges of the two 5-branes and $Q_{(3 \hookrightarrow 5)}$ and $Q'_{(3 \hookrightarrow 5)} = Q_{(3 \hookrightarrow 5')}$ are the charges of the 3-brane defects, respectively.

[8]In fact there is a third non-standard type of string intersection that of two M-5-branes intersecting at a string [18]. For other non-standard intersections see [27].

[9]The tension of M-branes in the conventions of [13] is $T_{(2)} = \mu_{(2)} = 2\pi m_p^3$ and $T_{(5)} = \mu_{(5)} = 2\pi m_p^6$, where m_p is the eleven-dimensional Plank mass.

To explore further the intersection of two M-5-branes on a 3-brane defect let us we assume that $Q_{(5)} = Q'_{(5)}$ and that the normal bundle $N_{(5)}$ of B_5 can be written as

$$N_{(5)} = E \oplus H\,, \tag{4.4}$$

where E is a rank two bundle and H is its compliment in $N_{(5)}$. Then the intersection can be described as follows: we first identify the M-5-brane B_5 as the image of the zero section of E while we identify the second M-5-brane B'_5 as the image of as a generic section s of E, i.e. $B'_5 = s(B_5)$. If B_5 and B'_5 are in general position, then they intersect transversaly in E. The 3-brane defect from the perspective of B_5 is the zero locus Z of the section s, i.e. $Z_3 = B_5 \cap s(B_5)$. The normal bundle of Z_3 in B_5 is E and in this case the Thom class of E can be identified with its Euler class $e(E)$. So the charge $Q_{(3\hookrightarrow 5)} = Q'_{(3\hookrightarrow 5)}$ can be identified with the Euler number of E in some units.

4.3 The 0-brane worldvolume soliton

The method of relating the charges of various worldvolume defects associated with brane intersections can be easily generalized to intersections that involve three or more branes. As an illustration we shall describe brane surgery for a configuration for which the associated orthogonal M-brane intersection is

$$\begin{array}{r} M5: 0,\ 1,\ \ 2,\ \ 3,\ \ 4,\ \ 5,\ \ -,\ -,\ - \\ M2: 0,\ 1,\ \ -,\ -,\ -,\ -,\ 6,\ \ -,\ - \\ M2': 0,\ -,\ 2,\ \ -,\ -,\ -,\ -,\ 7,\ \ -\ . \end{array} \tag{4.5}$$

We define $Z_1 = B_2 \cap B_5$, $Z_{1'} = B_{2'} \cap B_5$ and $Z_0 = Z_1 \cap Z_{1'} = B_2 \cap B_{2'} \cap B_5$. In such configuration the normal bundles of the M-branes involved in the intersection split as follows:

$$\begin{aligned} N_{(2)}|_{Z_0} &= N_{(1\hookrightarrow 5)}|_{Z_0} \oplus N_{(1'\hookrightarrow 2')}|_{Z_0} \oplus N_{(2+2'+5)}|_{Z_0} \\ N_{(2')}|_{Z_0} &= N_{(1'\hookrightarrow 5)}|_{Z_0} \oplus N_{(1\hookrightarrow 2)}|_{Z_0} \oplus N_{(2+2'+5)}|_{Z_0} \\ N_{(2)}|_{Z_0} &= N_{(0\hookrightarrow 2')} \oplus N_{(1+1'\hookrightarrow 5)}|_{Z_0} \oplus N_{(2+2'+5)}|_{Z_0} \\ N_{(2')}|_{Z_0} &= N_{(0\hookrightarrow 2)} \oplus N_{(1+1'\hookrightarrow 5)}|_{Z_0} \oplus N_{(2+2'+5)}|_{Z_0} \\ N_{(5)}|_{Z_0} &= N_{(1\hookrightarrow 2)}|_{Z_0} \oplus N_{(1'\hookrightarrow 2')}|_{Z_0} \oplus N_{(2+2'+5)}|_{Z_0}\,. \end{aligned} \tag{4.6}$$

In addition we have

$$\begin{aligned} N_{(1\hookrightarrow 5)}|_{Z_0} &= N_{(0\hookrightarrow 1')} \oplus N_{(1+1'\hookrightarrow 5)}|_{Z_0} \\ N_{(1'\hookrightarrow 5)}|_{Z_0} &= N_{(0\hookrightarrow 1)} \oplus N_{(1+1'\hookrightarrow 5)}|_{Z_0} \\ N_{(0\hookrightarrow 5)}|_{Z_0} &= N_{(0\hookrightarrow 1)} \oplus N_{(0\hookrightarrow 1')} \oplus N_{(1+1'\hookrightarrow 5)}|_{Z_0}\,. \end{aligned} \tag{4.7}$$

The first two of above three decompositions can be understood by viewing the strings Z_1 and $Z_{1'}$ as branes within the M-5-brane. The above decompositions of the normal bundles lead to the following relations amongst the brane charges:

$$
\begin{aligned}
Q_{(2)} &= Q_{(1\hookrightarrow 5)} Q_{(1'\hookrightarrow 2')} Q_{(2+2'+5)} \\
Q_{(2')} &= Q_{(1'\hookrightarrow 5)} Q_{(1\hookrightarrow 2)} Q_{(2+2'+5)} \\
Q_{(2)} &= Q_{(0\hookrightarrow 2')} Q_{(1+1'\hookrightarrow 5)} Q_{(2+2'+5)} \\
Q_{(2')} &= Q_{(0\hookrightarrow 2)} Q_{(1+1'\hookrightarrow 5)} Q_{(2+2'+5)} \\
Q_{(5)} &= Q_{(1\hookrightarrow 2)} Q_{(1'\hookrightarrow 2')} Q_{(2+2'+5)} \,,
\end{aligned}
\tag{4.8}
$$

and

$$
\begin{aligned}
Q_{(1\hookrightarrow 5)} &= Q_{(0\hookrightarrow 1')} Q_{(1+1'\hookrightarrow 5)} \\
Q_{(1'\hookrightarrow 5)} &= Q_{(0\hookrightarrow 1)} Q_{(1+1'\hookrightarrow 5)} \\
Q_{(0\hookrightarrow 5)} &= Q_{(0\hookrightarrow 1)} Q_{(0\hookrightarrow 1')} Q_{(1+1'\hookrightarrow 5)} \,.
\end{aligned}
\tag{4.9}
$$

Using the above relations, we find

$$
\begin{aligned}
Q_{(5)} Q_{(0\hookrightarrow 5)} &= Q_{(0\hookrightarrow 2)} Q_{(2)} \\
Q_{(5)} Q_{(0\hookrightarrow 5)} &= Q_{(0\hookrightarrow 2')} Q_{(2')} \,,
\end{aligned}
\tag{4.10}
$$

which leads to

$$
Q_{(5)} Q_{(0\hookrightarrow 5)} = \left(Q_{(0\hookrightarrow 2)} Q_{(0\hookrightarrow 2')} Q_{(2)} Q_{(2')}\right)^{1/2} . \tag{4.11}
$$

This relates the charges of the defect as measured on membranes and M-5-brane with the charges of membranes and M-5-brane involved in the configuration. The above computation can be easily extended to many other triple brane intersections.

5. Concluding remarks

We have proposed a method to investigate charge conservation at the intersection of two or more branes based on Thom classes. We have found that this has led to a relation between the charges of the branes involved in the intersection and those of the associated worldvolume defects. We have then explored these relations for various brane intersections in strings and M-theory.

Some brane intersections preserve a proportion of spacetime supersymmetry. This can be incorporated in our brane surgery construction by imposing additional restrictions on the spacetime and the submanifolds associated with the various branes. For example, one can introduce a supersymmetry projection operator at every point in the submanifold associated with a brane in a way similar to that of [19] and then ask whether there are killing spinors that satisfy all these projections. However as we have seen brane charge conservation can be investigated without the additional restriction of supersymmetry.

It is natural to ask whether there are solutions in the literature that satisfy all the requirements necessary to establish the above relations amongst the charges of the branes and those of the defects. This is related to the question of localization of the brane intersection solutions. It has been observed in the beginning of construction of supergravity solutions with the interpretation as brane intersections [4, 20] that they are smeared along their relative transverse directions and that their asymptotic behaviour does not have the desired power decay law with respect to some radial coordinate. The latter leads to problems for calculating the charges at infinity of the associated branes. Thus such intersections are geometrical and there is no defect on either brane involved in the intersection. Subsequent improvements in the solutions by adding different harmonic functions for each intersecting brane [17] have not resolve the problem. More recently solutions [21, 22, 23, 24] have been found using the so called generalized harmonic function equations first proposed in [25]. Such solutions exhibit a partial localization at the intersection. However, their associated form-field strengths do not have the desirable asymptotic behaviour required for the evaluation of brane charges. Because of these and other arguments, it was suggested in [26] that for many intersecting brane configurations there are not exit solutions which are completely localized. However it may simply be that supergravity solutions that exhibit charge conservation at brane intersections are not simply constructed from harmonic functions and their (straightforward) generalizations. So it appears that new methods should be developed to solve the supergravity field equations like those for BPS monopoles in gauge theories.

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References

[1] J. Dai, R.G. Leigh, and J. Polchinski, *New Connections between String Theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073.

[2] A. Strominger, *Open p-branes*, *Phys. Lett.* **B 383** (1996) 44 [hep-th/9512059].

[3] P.K. Townsend, *D-Branes from M-Branes*, *Phys. Lett.* **B 373** (1996) 68 [hep-th/9512062].

[4] G. Papadopoulos and P.K. Townsend, *Intersecting M-branes*, *Phys. Lett.* **B 380** (1996) 273 [hep-th/9603087].

[5] P.K. Townsend, *Brane Surgery*, *Nucl. Phys.* **58** *(Proc. Suppl.)* (1997) 163 [hep-th/960921].

[6] J. Polchinski, in *Fields, Strings and Duality* (Tasi '96) Ed. C. Efthimiou and B. Greene, World Scientific, 1997, hep-th/9611050.

[7] E. Witten, *Five-brane Effective Action in M-theory*, *J. Geom. Phys.* **22** (1997) 103 [hep-th/9610234].

[8] G.W. Gibbons, G.T. Horowitz and P.K. Townsend, *Higher Dimensional Resolution of Dilatonic Black Hole Singularities*, *Class. and Quant. Grav.* **12** (1995) 297 [hep-th/9410073].

[9] R. Bott and L.W. Wu, *Differential Forms in Algebraic Topology*, Springer-Verlag, Berlin, 1982.

[10] J. Schwarz, *Lectures on Superstrings and M-theory Dualities*, *Nucl. Phys.* **B55** *(Proc. Suppl.)* (1997) 1 [hep-th/9607201].

[11] C.G. Callan and J.M. Maldacena, *Brane Dynamics from the Born-Infeld Action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

[12] E. Bergshoeff, J.P. van der Schaar and G.Papadopoulos, *Domain Walls on the Brane*, *Phys. Lett.* **B 430** (1998) 63 [hep-th/9801158].

[13] J.H. Schwartz, *From Superstrings to M-theory*, hep-th/9807135.

[14] J.H. Schwartz, *The power of M-theory*, *Phys. Lett.* **B 367** (1996) 97 [hep-th/9510086]; *M-Theory extensions of T-duality*, hep-th/9601077.

[15] I.V. Lavrinenko, H. Lü, C.N. Pope and K.S. Stelle, *Superdualities, Brane Tensions and Massive IIA/IIB Duality*, hep-th/9903057.

[16] G. Papadopoulos, *T-Duality and the Worldvolume Solitons Solitons of Five-Branes and KK-Monopoles*, *Phys. Lett.* **B 434** (1998) 277 [hep-th/9712162].

[17] A.A. Tseytlin, *Harmonic Superposition of M-Branes*, *Nucl. Phys.* **B 475** (1996) 149 [hep-th/9604035].

[18] J.P. Gauntlett, D.A. Kastor and J. Traschen, *Overlapping Branes in M-theory*, *Nucl. Phys.* **B 478** (1996) 544 [hep-th/9604179].

[19] K. Becker, M. Becker and A. Strominger, *Fivebranes, Membranes and non-perturbative string theory*, *Nucl. Phys.* **B 456** (1995) 130 [hep-th/9507158];
E. Bergshoeff, R. Kallosh, T. Ortin and G. Papadopoulos, *Kappa-Symmetry, Supersymmetry and Intersecting Branes*, *Nucl. Phys.* **B 502** (1997) 149 [hep-th/9705040].

[20] G. Papadopoulos, *A Brief Guide to p-Branes*, in *Recent Developments in Theoretical Physics: STU-Dualities and Nonperturbative Phenomena in Superstrings and Supergravity*, *Forch. Phys.* **44** (1996) 573 [hep-th/9604068].

[21] H. Yang, *Localized Intersecting Brane Solutions of D=11 Supergravity*, hep-th/9902128.

[22] D. Youm, *Localized Intersecting BPS Branes*, hep-th/9902208.

[23] A. Fayyazuddin and D.J. Smith, *Localized intersections of M5-branes and four-dimensional superconformal field theories*, *J. High Energy Phys.* **04** (1999) 030 [hep-th/9902210].

[24] A. Loewy, *Semi Localized Brane Intersections in SUGRA*, hep-th/9903038.

[25] J.P. Gauntlett, G.W. Gibbons, G. Papadopoulos and P.K. Townsend, *Hyper-Kähler Manifolds and Multi-Intersecting Five Branes*, *Nucl. Phys.* **B 500** (1997) 133 [hep-th/9702202].

[26] D. Marolf and A. Peet, *Brane Baldness vs Superselection Sectors*, hep-th/9903213.

[27] V.D. Ivashchuk and V.N. Melnikov, *Madjumbar-Papapetrou type solutions in sigma models and intersecting branes*, *Class. and Quant. Grav.* **16** (1999) 849 [hep-th/9802121];
M.A. Grebeniuk and V.D. Ivashchuk, *Sigma model solutions and intersecting p-branes related to the Lie Algebras*, *Phys. Lett.* **B 442** (1998) 125 [hep-th/9805113].